

Chapter 18

- Protons and electrons, which are the foundation of all matter, have *electric charge*.
 - The smallest unit of charge is $e = 1.6 \times 10^{-19}$ C (in Coulombs).
 - An electron has charge $-e$ and mass m_e , and a proton has charge $+e$ and mass m_p .
 - Any quantity of charge, q , comes in units of e : $q = (\# \text{ of electrons or protons}) \times e$

LIKE CHARGES REPEL.

\Rightarrow This *force* between charges is mathematically characterized by **Coulomb's Law**:

$$\vec{F} = \frac{kq_1q_2}{r^2}$$

- UNLIKE CHARGES ATTRACT.

which has a magnitude and a direction (is a vector) and is measured in Newtons (N).

- Charged matter (either positive or negative) creates an **electric field**, \vec{E} , given by $\vec{E} = \frac{kq}{r}$, which is also a vector and has units of N/C or V/m . And, playing with the equations, we get:

$$\vec{F} = q\vec{E}.$$

- The direction of the electric field, \vec{E} , is *defined* as the direction a positive “test” charge will go.
- We *visualize* this electric field, \vec{E} , via the use of **electric field lines**, which:
 - Begin on positive charge.
 - End on negative charge.
 - The number of field lines you draw is arbitrary, but the number must be *proportional to the charge* (e.g. if you choose 4 field lines to begin on a positive charge of $+q$, then 8 must begin on a positive charge of $+2q$: $\frac{1 \text{ charge}}{4 \text{ lines}} = \frac{2 \text{ charges}}{8 \text{ lines}}$).
 - Electric field lines *never overlap*.
- Conductors: materials where charge is free to move easily.
 - Charge will always go to the surface of a conductor and spread out evenly (remember, like charges repel).
 - When two conductors touch (this is called induction), the charges mix, i.e., the final charge on both afterwards will be the same and equal to: $q_f = (q_1 + q_2)/2$ (including sign).
 - Electric field lines always begin or end perpendicular to a conductor's surface.
- Notes on solving problems:
 - \vec{F} and \vec{E} are vectors, so you have to add individual forces or electric fields vectorially. For example, if you have 3 charges and you're asked to calculate the electric field at some point in space, calculate \vec{E}_1 , \vec{E}_2 , and \vec{E}_3 separately then add them up (with x and y components, you know the drill: $\vec{E}_{\text{tot}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$).
 - Don't do too much work: once you have the electric field at a point, \vec{E}_{tot} , you can calculate the force on a charge, q , placed at that point with just $\vec{F} = q\vec{E}$ (still in the same direction as \vec{E}).
 - Determine the direction of \vec{F} and \vec{E} with your knowledge of physics (likes repel, unlikes attract) then solve with the magnitudes with the equations.
- Gauss's Law: Gauss's law is easy. It allows you to calculate the electric flux, Φ , and the magnitude of the electric field, E , going through some *imaginary* surface A .

$$\Phi = EA = Q_{in}/\epsilon_0$$

- Q_{in} is the charge inside A —and that's it—it doesn't matter what shape it is...
- For example, if your imaginary surface is a spherical surface of $A = 4\pi r^2$ and inside it is a metal ball of radius $R < r$ and charge Q : it doesn't matter one bit how big the ball is, except that it's entirely in A and has charge Q . So $Q_{in} = Q$ and your problem is solved.
- If you have oppositely charged things inside A , just add them (with signs).
- If Q_{in} is positive, the electric field points out. If Q_{in} is negative, the electric field points in.

Chapter 19

- Electric potential energy and electric potential
 - Just as gravitational potential energy (mgh) is associated with the conservative force of gravity ($\vec{F}_G = m\vec{g}$), **electric potential energy**, EPE, is associated with the electrostatic force ($\vec{F}_E = q\vec{E}$).
 - Just as charge generates an electric field, \vec{E} , so to does it generate an **electric potential**, $V = \frac{kq}{r}$.
 - * The potential, V , depends only on distance—it's just a scalar not a vector.
 - * This means that V is just a number (with no direction). So, for example, if you want to calculate the potential V_{tot} at a point in space due to three charges q_1 , q_2 , and q_3 distances r_1 , r_2 , and r_3 away, all you need to do is add!: $V_{\text{tot}} = V_1 + V_2 + V_3 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3}$.
 - * $EPE = qV$
- The change in energy needed to do something is equal to the work needed to do that: $\Delta E = W$.
- So the work done by the electric force is $W_{AB} = EPE_A - EPE_B$ from the point A to B .
 - If the work done by the electric force, W , is positive, then energy is gained in the process (the system wanted to do that).
 - If W is negative, the system didn't want to do that, it *required* energy.
- Notes: Just as with \vec{E} and \vec{F} , don't do too much work. Once you have V , EPE is just qV .
- Remember:
 - Power = energy/time
 - $F = ma$
 - $E_{\text{kinetic}} = \frac{1}{2}mv^2$, $E_{\text{potential}} = mgh$
 - Conservation of energy: $E_{\text{initial}} = E_{\text{final}}$
- **Equipotential lines** (think equal-potential) are regions (lines, surfaces) where V is the same.
 - They're always perpendicular to \vec{E} -field lines.
 - They're always loops; they never end or begin.
- A relation between E and V : $\vec{E} = -\frac{\Delta V}{\Delta s}$
- Capacitors:
 - The electric field between two parallel plates of area A and charge q is uniform and points from the positive side to the negative side.
 - q is the charge on one of the plates; $-q$ is the charge on the other plate.
 - $q = CV$ where C is the capacitance (measured in farads, F).
 - Dielectrics:
 - * Add a dielectric, κ , to a capacitor and the electric field between the plates decreases; $\kappa = \frac{E_f}{E_d}$.
 - * The capacitance is given by: $C = \frac{\kappa\epsilon_0 A}{d}$. Without the dielectric (vacuum), $\kappa = 1$.
 - * Energy of a capacitor: $E = \frac{1}{2}CV^2$.

- Electric Circuits

- When a circuit is connected to a power supply, a potential difference V (: one side plus, one side minus) causes positive charges to move away from the + end: this causes a current to flow,

$$I = \frac{\Delta q}{\Delta t},$$

equal to the amount of charge going through a slice of the wire over some period of time.

- Current, I , is measured in amperes (or, in slang, amps), $A = C/s$.
- A battery (with an **electric motive force**, emf) or a **dc** (direct current) **power supply** produce a constant voltage and thus a constant current (dc current).
 - * In practice, \mathcal{E} is the same thing as V (it's just the V for a battery) and it works in all the equations you know just like V does: $\mathcal{E} = IR$, etc.
- The wall outlets and other **ac power supplies** produce a voltage that varies in time and looks like a $\sin()$ wave (this results in ac current).
 - * To make sense of this we speak of the average or “root mean square” voltage, $V_{\text{rms}} = V_0/\sqrt{2}$, which equals the peak voltage V_0 (the top of the sine wave) over the square root of two.
 - * For ac circuits ($V = V_0 \sin 2\pi ft$), use the same equations, just sub in V_{rms} for V .

- The golden rule of circuits: **Ohm's Law:** $V = IR$

- The voltage drop or voltage across or voltage difference across (they all mean the same thing save for maybe a minus sign) a resistor is equal to the current going through the resistor times its resistance.

- The resistance R of a material can be calculated with $R = \rho \frac{L}{A}$ where $\rho \equiv$ resistivity ($\Omega \cdot m$), $L =$ length, and $A =$ cross sectional area.
 - Resistance can also depend on temperature: $\rho = \rho_0[1 + \alpha(T - T_0)]$ where α is the **temperature coefficient of resistivity** for the particular material in question.
 - * If α is positive, the resistance, R , increases with temperature, T .
 - * If α is negative, R decreases with T .

- Electric Power: $P = IV = I^2R = V^2/R$

- Usually, in a circuit, the voltage is held constant. So if you're comparing two circuits both driven by a battery at +10V by looking at the ratio P_1/P_2 , the V 's are the same so they cancel.

- Again, for ac circuits:

$$* \text{ The average power, } \bar{P} = I_{\text{rms}} V_{\text{rms}} = \frac{I_0}{\sqrt{2}} \frac{V_0}{\sqrt{2}} = \frac{1}{2} I_0 V_0$$

- Reducing circuits with resistors only.

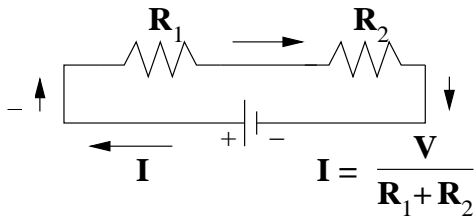
- All circuits with just resistors and *one* power supply can be reduced to a circuit with the same one power supply and one resistor of equivalent resistance, R_{eq} .

- To find R_{eq} , work from the inside out using the resistor combination rules:

$$* \text{ series resistors: } R_{\text{eq}} = R_1 + R_2 + \dots$$

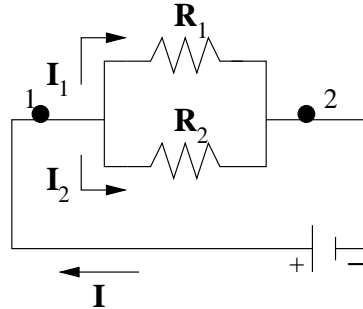
$$* \text{ parallel resistors: } R_{\text{eq}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots}$$

- More difficult circuits.



The current going through resistors in series *is the same*.

When a wire splits though, so can the current...Some of the current, I , in the circuit to the right will go through the top— I_1 —some of it will go through the bottom— I_2 .



- * To solve a simple parallel problem like this one, you have to realize that the potential drop across both R_1 and R_2 is the same.
- * So the voltage difference between points 1 and 2 equals the voltage across R_1 and across R_2 .
- * Then, if you know $V_{12} = V$, R_1 and R_2 , you're in business:

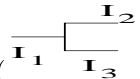
$$\text{(in this case)} \quad V_{12} = V = I_1 R_1 = I_2 R_2 \Rightarrow \frac{I_1}{I_2} = \frac{R_2}{R_1}$$

- * What does this tell us? It tells us that *current always goes through the path of least resistance*.

- Even more difficult circuits: multiple power supplies scattered God knows where.

– You must use **Kirchhoff's Rules**:

- * *Loop Rule*: The sum of all voltage gains and drops equals zero around a “closed loop” (meaning you start and stop at the same place - and no figure 8s) in a circuit.

- * *Junction Rule*: The sum of the currents going into a junction () equals the sum of the currents goint out. (In this case, $I_1 = I_2 + I_3$.)

- * The METHOD:

Step 1: Identify the different wire segments in the circuit and label them I_1, I_2, \dots on the circuit diagram (they each will have a different current which you will eventually solve for). (A wire segment goes from junction to junction without any splits.)

Step 2: Try to guess the direction of I_1, I_2, \dots and draw it on your diagram with an arrow, this is important. When guessing, remember:

- current goes from + to -,
- if you choose wrong, it's OK—you will know at the end because one of your I 's will be negative (you can just switch the arrow then, everything else will still be right).

Step 3: Find enough equations to match the number of currents (e.g. 3 equations for I_1, I_2 , and I_3).

- Use the loop and junction rules. Use the junction rule (pick any junction) for 1 equation, then use the loop rule for the other equations.

i) Pick your loops: a circle, no figure 8s.

ii) Start at a point on your loop, move clockwise.

→ When you come to a *power source*: if you're going from - to +, you gain voltage: write down “+V”; if you're going from + to -, you lose voltage: write down “-V”.

→ When you come to a *resistor, R_i* : if you're going **in the direction of the current**, $I_{i=1,2,or\dots}$, you **lose** voltage (voltage drop) equal to $I_i R_i = V_{\text{drop}}$: write down “ $-I_i R_i$ ”; if you're going **against the current**, you **gain** voltage, $V_{\text{gain}} = I_i R_i$: write down “ $+I_i R_i$ ”.

→ When you get back to where you started, you're done, write down “= 0”.

Step 4: Solve for the current you need: use substitution and a lot of algebra.

- Notes on problem solving:
 - Redraw complicated circuits in simpler form.
 - Kirchhoff's rules and Ohm's law *don't work for capacitors*. Don't even try.
- Reducing circuits with capacitors only.
 - Capacitor combination works exactly reverse to resistor combination:
 - * series capacitors: $C_{\text{eq}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \dots}$
 - * parallel capacitors: $C_{\text{eq}} = C_1 + C_2 + \dots$
- RC circuits (real brief)
 - A resistor, R, and a capacitor, C, are both in a circuit. It takes time to charge up or discharge the capacitor. The goal here is to figure the percentage of charge q/q_0 left on the capacitor at any given time, t .
 - capacitor charging: The capacitor is gaining charge eventually to reach some maximum value $q = CV$. The percentage will increase as time proceeds (t gets bigger).

$$\frac{\text{final charge}}{\text{initial charge}} = \frac{q}{q_0} = 1 - e^{-t/RC}$$

- capacitor discharging: The capacitor is losing charge eventually to reach $q = 0$. The percentage decreases as time proceeds.

$$\frac{\text{final charge}}{\text{initial charge}} = \frac{q}{q_0} = e^{-t/RC}$$