Electromagnetic (EM) waves: $\vec{E} \perp \vec{B}$

- The speed of a wave: $v = f\lambda$ where v is the velocity (m/s), f is the frequency of the wave (Hz or 1/s), and λ is the wavelength (m or nm($\times 10^{-9}$ m)).
- EM waves in a vacuum (or air) travel at the speed of light, $c = 3 \times 10^8 m/s = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$. [In other materials they travel at c/n where n is the index of refraction.]
- The magnitude of E and B are related: $|E| = c|B|$.
- EM waves (otherwise known as light) have the following properties:
  - they have an intensity, $S = cu = c\epsilon_0 E^2 = \frac{c}{\mu_0} B^2$ where $S = \text{intensity} = \frac{\text{total energy}}{\text{time}\times\text{Area}}$, $u = \text{total energy density} = (1/2)\epsilon_0 E^2 + (1/2\mu_0)B^2$, and E and B are the magnitudes of the electric and magnetic fields.
    - so the intensity is proportional to E^2.
  - the power, P, delivered by an EM wave is $P = Et = SA$, where E = energy!, t = time, S = intensity, and A = area of light beam.
  - also remember $E_{rms} = \frac{E_0}{\sqrt{2}}$ and $B_{rms} = \frac{B_0}{\sqrt{2}}$

Polarizors/Analyzers/Malus's Law All you need to know:

1. If you start with unpolarized light (either with peak values E_0, S_0 or rms values E_{rms}, S_{rms}) which goes through a polarizing sheet polarized θ from the vertical, then the light emerging from the sheet will be half as intense: $S_{final} = S_{initial}/2$, polarized in the θ from vertical direction, and with electric field magnitude $E_{final} = \sqrt{S_{final}/(c\epsilon_0)}$ (or with B = cE).
2. If you start with light polarized in the θ_1 from vertical direction which goes through a sheet polarized in the θ_2 direction, then use Malus's Law to find the final intensity: $S_{final} = S_{initial}[cos(\theta_2 - \theta_1)]^2$, then find E or B with that.
3. If you start with light polarized in the θ_1 direction which goes through a polarizing sheet also in the θ_1 direction, nothing happens.
4. Use the last two multiple times for multiple analyzer setups.

Dopplar Shift Usually plub and chug problems:

$$f' = f(1 \pm \frac{u}{c})$$

where f' is the observed frequency, f is the emitted frequency, + is for objects moving toward each other, − is for objects moving away from each other, u is the relative speed of the two moving objects, and c is the speed of light.

Lens and Mirror stuff, minimal tips and tricks

- You can do it all with:

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

  where f is the focal length (*negative for diverging lenses and convex mirrors; ∞ for plane mirrors*), d_i is the distance to the formed image, and d_o is the distance to the object,

  and with

$$m = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$$

  where m is the magnification, h_i is the image height, and h_o is the object height.

- For multiple lenses/mirrors in combination just use the image created by the first lens as the object for the second lens and so on ... and the total magnification is just the multiple of the magnifications for the individual optics elements: $m = m_1 \times m_2 \times ...$ The final height then is just m times the initial height: $h_{final} = mh_{initial}$.
- For spherical mirrors/lenses the radius of curvature is twice the focal length: $f = (1/2)R$.

Ray Diagrams

- Use the equations first if you can and figure out where the image/object should go.
- You need two rays two intersect to find where the image is created.
- Tips:
  1. Think of objects as light bulbs emitting light in all directions.
  2. Draw the rays you know:
  3. for lenses:
     - If a ray goes through the center of a lens, if will just keep on going without being deflected.
     - *converging lens*: If a ray goes through a focal point on one side of the lens, it will be straight on (parallel) on the other side.
     - *diverging lens*: If a ray goes through a diverging lens parallel to the center line, it will be deflected away from the center such that if you draw a dotted line back from the deflected ray, it will intersect the focal point.
     - If rays don't converge on the opposite side of an object, then you have a virtual image (use the dotted line image to get converging lines on the same side as the object).
  4. for mirrors:
     - Rays going through the radius of curvature point (R) (either in a concave mirror or with "dots" as in a convex mirror) bounce back on themselves.
     - *concave mirrors:* Rays that go through the focal point reflect parallel to the center line. Rays that come in parallel are reflected through the focal point.
     - *convex mirrors:* In a way, the same goes for convex mirrors: only here the focal point is behind the mirror and the rays are reflected away from the center line (but the dotted line goes through the focal point. (**All images created by a convex mirror are virtual.**)

Real/Virtual, Plus/Minus, Right/Wrong

Figuring out if an image is real or virtual can be a pain, especially in multi-optics situations. Here are some tips:

1. Generally, minus signs for d_i mean virtual.
2. I subscribe to the good vs. bad, right vs. wrong philosophy of optics. If the image is on the wrong side (meaning it shouldn't be where it is) then it's virtual. So what is the right or good side?
   - For lenses: **A real image should be on the opposite side of the object**. The object emits light, it goes through the lens, and an image is formed on the other side which will appear on a piece of paper if you put one there. If the object and the image distances are on opposite sides of each other, then they are both positive. Having a real image on the same side of the lens as and the object is wrong (light doesn't reflect off a perfect lens) and virtual.
   - For lenses. **A real image appears on the same side of the object**. The object emits light, it's reflected off the mirror, and an image is formed on the same side as the object. Having a real image behind a mirror is wrong (light can't go through a mirror!) and virtual. If the object and the image distances are on the same side, then they are both positive.

Snell's Law and Brewster's law

The direction of a light ray going from one medium (with index of refraction n) to another is given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where n_1 is the index of refraction for the incident medium, θ_1 is the angle of incidence *relative to the normal(perpindicular) of the surface*, n_2 is for the second medium, and θ_2 is the angle of reflection relative to the normal *on the other side of the interface.*

Total internal reflection occurs when $\theta_2 = 90^{\cdot}$ and the reflected ray is bounced back into medium 1.

The **Brewster angle** occurs when the angle of incidence (now given by θ_B) across an n_1—n_2 interface results in reflected light which is completely polarized:

$$\tan\theta_B = \frac{n_1}{n_2}.$$

Interference

- Double slit:
  - **bright fringes:** $sin\theta = m\frac{\lambda}{d}$ where m = 0, 1, 2, ... is the order of the fringe (0 being at the center) and *d is the distance between the slits* and theta is the angle to the fringe in question.
  - dark fringes: $sin\theta = (m + \frac{1}{2})\frac{\lambda}{d}$ where m = 0, 1, 2, ... is the order of the fringe (0 being at the center) and *d is the distance between the slits* and theta is the angle to the fringe in question.
  - To find the distance from the center to whatever fringe, y, just use geometry: $\tan\theta = \frac{y}{L}$
- Single slit:
  - Single slits act **the opposite of double slits**.
  - **dark fringes:** $sin\theta = m\frac{\lambda}{W}$ where m = 1, 2, ... is the order of the fringe (there is no zeroth dark fringe) and *W is the width of the slit* and theta is the angle to the dark fringe in question. There is no zeroth dark fringe.
- Resolving power: $\sin\theta = 1.22\frac{\lambda}{D}$ where D is the distance of the opening (e.g. the pupil), λ is the wavelength of the light, and θ, like above, let's you determine the minimum distance between the two objects ($\tan y \approx y \approx \theta L$).
- Diffraction grating:
  - the equations work just like the double slit only *you can only find maximum fringes not dark fringes.*
  - $\sin\theta = m\frac{\lambda}{d}$ where m = 0, 1, 2, ..., and d is the distance between lines on the grating.
  - $d = \frac{1}{\text{\# of lines per meter}}$ or # of lines per meter = $\frac{1}{d}$